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THE STABILITY OF PERMANENT ROTATIONS OF A TOP WITH A CAVITY FILLED WITH LIQUID ON A PLANE WITH FRICTION†

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The problem of the motion of a heavy dynamically symmetrical sphere along a horizontal plane with friction is considered. Inside the sphere there is an axisymmetric ellipsoidal cavity, completely filled with an ideal incompressible fluid, which performs uniform rotational motion. It is shown that the system, in addition to an integral of the constancy of vortex intensity, allows of a Jellet type integral. In addition, the total mechanical energy of the system is a non-increasing function. The stability of permanent rotations of the system is investigated using a modified Routh theorem [1, 2]. Special cases of a spherical cavity and a weightless envelope are considered. © 2000 Elsevier Science Ltd. All rights reserved.

1. FORMULATION OF THE PROBLEM

Suppose a dynamically symmetrical sphere with an axisymmetrical ellipsoidal cavity, completely filled with an ideal incompressible fluid, executing uniform rotational motion, moves along a horizontal plane. Unlike the case considered previously of an absolutely smooth plane [3], we will assume that the plane is rough, i.e. the reaction of the plane is the sum of the the normal reaction and the force of sliding friction.

We will assume that the axis of symmetry of the cavity coincides with the axis of dynamic symmetry of the sphere, while the centre of mass G of the system is a distance c from the geometrical centre of the sphere. Suppose $Gx_1x_2x_3$ are the principal axes of inertia of the system, and the positive direction of the axis of symmetry Gx_3 is chosen so that the z-coordinate of the geometrical centre of the sphere is equal to $c > 0$. With these conditions the equation of the cavity has the form.

$$x_1^2/a_1^2 + x_2^2/a_2^2 + (x_3 - a)^2/a_3^2 = 1$$

where $a_1 = a_2$ and a_3 are the semi-axes of the cavity, and a is the z-coordinate of its geometrical centre (in particular, in the case of a weightless envelope $a = 0$).

The equations of motion of the system, referred to the system of coordinates $Gx_1x_2x_3$, have the form [1, 2]

$$m(\dot{v}_1 + \omega_2 v_3 - \omega_3 v_2) = -mg\gamma_1 + R_1 \quad (123) \tag{1.1}$$

$$A_{*1}\dot{\omega}_1 + A_1'\dot{\Omega}_1 + (A_{*3} - A_{*2})\omega_2\omega_3 + A_3'\omega_2\Omega_3 - A_2'\omega_3\Omega_2 = \rho_2 R_3 - \rho_3 R_2 \quad (123) \tag{1.2}$$

$$\dot{\Omega}_1 + 2a_1^2\left(\frac{(\omega_2 - \Omega_2)\Omega_3}{a_3^2 + a_1^2} - \frac{(\omega_3 - \Omega_3)\Omega_2}{a_1^2 + a_2^2}\right) = 0 \quad (123) \tag{1.3}$$

$$\dot{\gamma}_1 + \omega_2\gamma_3 - \omega_3\gamma_2 = 0 \quad (123) \tag{1.4}$$

Here

$$A_1^* = \frac{m_l(a_2^2 - a_3^2)^2}{5(a_2^2 + a_3^2)}, \quad A_1' = \frac{4m_l a_2 a_3^2}{5(a_2^2 + a_3^2)} \quad (123)$$

$$A_{*i} = A_i + A_i^*, \quad m = m_b + m_l$$

The symbol (123) denotes that the two unwritten relations are obtained from the written relation by cyclic permutation of the indices 1, 2, 3, m_b is the mass of the envelope, m_1 is the mass of the fluid, A_1,

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A_2 and A_3 are the principal central moments of inertia of the envelope ($A_1 = A_2$), v_i and ω_i, Ω_i, γ_i, R_i, and ρ_i denote the projections of the vectors $\mathbf{v}$, $\boldsymbol{\omega}$, $\boldsymbol{\Omega}$, $\boldsymbol{\gamma}$, $\mathbf{R}$ and $\boldsymbol{\rho}$ onto the $Gx_1x_2x_3$ axes, $\mathbf{v}$ is the velocity of the centre of the mass, $\boldsymbol{\omega}$ is the angular velocity, $2\boldsymbol{\Omega}$ is the vortex vector, $\boldsymbol{\gamma}$ is the unit vector of the ascending vertical, $\mathbf{R} = N\boldsymbol{\gamma} + \mathbf{F}$ is the reaction of the plane, ($\mathbf{F}$ is the friction force: $\mathbf{F} \cdot \boldsymbol{\gamma} = 0$, $\mathbf{F} \cdot (\mathbf{v} + \boldsymbol{\omega} \times \boldsymbol{\rho}) \leqslant 0$, $\boldsymbol{\rho} = c\mathbf{e}_3 - r\boldsymbol{\gamma}$ is the radius vector of the point where the top touches the reference plane with respect to the centre of mass ($\mathbf{e}_3$ is the unit vector of the Gx_3 axis and r is the radius of the spherical envelope)).

Equation (1.1) expresses the theorem of the change in the momentum of the system, Eq.(1.2) expresses the theorem on the change in the kinetic momentum, Eq.(1.3) expresses Helmholtz theorem, while Eq.(1.4) expresses the condition for the vector $\boldsymbol{\gamma}$ to be constant in the inertial system of coordinates. To obtain a closed system of equations we must specify some of friction law in the form $\mathbf{F} = \mathbf{F}(N, \mathbf{v}, \boldsymbol{\omega}, \boldsymbol{\Omega}, \boldsymbol{\gamma})$ and supplement Eqs.(1.1)–(1.4) by the equation

$$(\mathbf{v} + \boldsymbol{\omega} \times \boldsymbol{\rho}) \cdot \boldsymbol{\gamma} = 0 \tag{1.5}$$

which expresses the condition that the top always remains in contact with the plane.

2. PERMANENT ROTATIONS AND THEIR STABILITY

It can be shown that the total mechanical energy H of the top does not increase by virtue of system (1.1)–(1.5), and this system allows of three first integrals – the generalized Jellet integral J, the integral of constancy of the vortex intensity W and the geometrical integral Γ

$$H = \frac{1}{2}m(v_1^2 + v_2^2 + v_3^2) + \frac{1}{2}(A_{*1}\omega_1^2 + A_{*1}\omega_2^2 + A_{*3}\omega_3^2) + \frac{1}{2}(A_1'\Omega_1^2 + A_1'\Omega_2^2 + A_3'\Omega_3^2) - \\ -mgc\gamma_3 \leqslant h(\dot{H} \leqslant 0) \tag{2.1}$$

$$J = (A_{*1}\omega_1 + A_1'\Omega_1)\gamma_1 + (A_{*1}\omega_2 + A_1'\Omega_2)\gamma_2 + (A_{*3}\omega_3 + A_3'\Omega_3)\gamma_3 - \varepsilon A_{*3}\omega_3 = k \quad (\varepsilon = c/r) \tag{2.2}$$

$$W = \Omega_1^2 + \Omega_2^2 + \delta^2\Omega_3^2 = \Omega^2 \quad (\delta = a_1/a_3) \tag{2.3}$$

$$\Gamma = \gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1 \tag{2.4}$$

By the generalized Routh theory [1, 2] steady motions of the system correspond to critical points of the increasing function H at fixed levels of the first integrals J, W and Γ, where the minimum points are stable steady motions.

We will solve the problem of finding the critical points of the function H in two stages. We will first obtain [2] the sole minimum of the function (2.1) with respect to the variable $\mathbf{v}$ and $\boldsymbol{\omega}$ at a fixed level of the Jellet integral (2.2), by considering the variables $\boldsymbol{\Omega}$ and $\boldsymbol{\gamma}$ as parameters. To do this we will introduce the function $\Phi = H - \lambda(J - k)$, where λ is an undetermined Lagrange multiplier, and we will write the conditions for it to be steady with respect to the variables $\mathbf{v}$, $\boldsymbol{\omega}$ and λ

$$\frac{\partial \Phi}{\partial \mathbf{v}} = m\mathbf{v} = \mathbf{0} \tag{2.5}$$

$$\frac{\partial \Phi}{\partial \omega_i} = A_{*i}(\omega_i - \lambda(\gamma_i - \delta_{i3}\varepsilon)) = 0, \quad i = 1,2,3 \quad (A_{*1} = A_{*2}) \tag{2.6}$$

$$\frac{\partial \Phi}{\partial \lambda} = -(J - k) = 0 \tag{2.7}$$

(δ_{ij} is the Kronecker delta).

It follows from Eqs (2.5) and (2.6) that

$$\mathbf{v} = \mathbf{0}, \quad \omega_i = \lambda(\gamma_i - \delta_{i3}\varepsilon), \quad i = 1, 2, 3 \tag{2.8}$$

Substituting expressions (2.8) into Eq.(2.7), we obtain

$$\lambda = \Lambda/\Delta_\varepsilon, \quad \Lambda = k - (A_1'\Omega_1\gamma_1 + A_1'\Omega_2\gamma_2 + A_3'\Omega_3\gamma_3), \quad \Delta_\varepsilon = A_{*1}(\gamma_1^2 + \gamma_2^2) + A_{*3}(\gamma_3 - \varepsilon)^2 \tag{2.9}$$

Hence (see (2.1), (2.8) and (2.9)),

$$\min_{\mathbf{v},\boldsymbol{\omega}} H|_{J=k} = V_k = \frac{1}{2}(A_1'\Omega_1^2 + A_1'\Omega_2^2 + A_3'\Omega_3^2) - mgc\gamma_3 + \frac{1}{2}\frac{\Lambda^2}{\Delta_\varepsilon} \tag{2.10}$$

We can interpret the function $V_k(\mathbf{\Omega}, \boldsymbol{\gamma})$ as the generalized effective potential. Its critical points on the direct product of the ellipsoid (2.3) and the sphere (2.4) correspond (taking relations (2.8) and (2.9) into account) to steady motions of the system, where the minimum points are stable steady motions.

To find the critical points of function (2.10) with conditions (2.3) and (2.4) we introduce the function

$$\Psi = V_k - \frac{1}{2}\mu(W - \Omega^2) - \frac{1}{2}\nu(\Gamma - 1)$$

where μ and ν are undetermined Lagrange multipliers, and we can write the conditions for it to be stationary with respect to the variables $\mathbf{\Omega}$, $\boldsymbol{\gamma}$, μ and ν as

$$\begin{aligned}
&\frac{\partial\Psi}{\partial\Omega_i} = -\frac{\Lambda}{\Delta_\varepsilon}A_i'\gamma_i + A_i'\Omega_i - \mu[1 + \delta_{i3}(\delta^2 - 1)]\Omega_i = 0\\
&\frac{\partial\Psi}{\partial\gamma_i} = -\frac{\Lambda}{\Delta_\varepsilon}A_i'\Omega_i - \frac{\Lambda^2}{\Delta_\varepsilon^2}A_{*i}(\gamma_i - \delta_{i3}\varepsilon) - \nu\gamma_i - mgc\delta_{i3} = 0, \quad i = 1,2,3\\
&\frac{\partial\Psi}{\partial\mu} = -\frac{1}{2}(W - \Omega^2) = 0, \quad \frac{\partial\Psi}{\partial\nu} = -\frac{1}{2}(\Gamma - 1) = 0
\end{aligned} \tag{2.11}$$

The system of equations (2.11) allows of the solutions

$$\begin{aligned}
&\gamma_1 = \gamma_2 = \Omega_1 = \Omega_2 = 0, \quad \gamma_3 = \pm 1, \quad \Omega_3 = \Omega/\delta\\
&\left(\mu_\pm = \frac{A_3'}{\delta^2} - A_3'\frac{\omega_\pm}{\Omega\delta(1 \mp \varepsilon)}, \quad \omega_\pm = \frac{k \mp A_3'\Omega/\delta}{A_{*3}(1 \mp \varepsilon)}\right.\\
&\left.\nu_\pm = \mp mgc - A_3'\frac{\omega_\pm\Omega}{\delta(1 \mp \varepsilon)} - A_{*3}\frac{\omega_\pm^2}{(1 \mp \varepsilon)}\right)
\end{aligned} \tag{2.12}$$

The upper (lower) sign in the expression for γ_3 corresponds to the upper (lower) sign in all the other expressions of relations (2.12).

These solutions correspond to permanent rotations of the rigid body (the envelope) and the fluid around a vertically situated axis of dynamic symmetry for the highest ($\gamma_3 = -1$) and the lowest ($\gamma_3 = +1$) positions of the mass centre

$$\begin{aligned}
&\gamma_1 = \gamma_2 = \Omega_1 = \Omega_2 = \omega_1 = \omega_2 = v_1 = v_2 = v_3 = 0\\
&\gamma_3 = \pm 1, \quad \omega_3 = \omega_\pm, \quad \Omega_3 = \Omega/\delta
\end{aligned} \tag{2.13}$$

To investigate the stability of solutions (2.13) it is sufficient to analyse the second variation $\delta^2\Psi$ of the function Ψ on the linear manifold $\delta W = \delta\Gamma = 0$, which for these solutions has the form

$$\delta\Omega_3 = \delta\gamma_3 = 0 \tag{2.14}$$

For solution (2.13) in the case when $\gamma_3 = +1$ we obtain

$$\delta^2\Psi=\frac{1}{2}\sum_{j=1}^{2}[A(\delta\Omega_j)^2+2B\delta\Omega_j\delta\gamma_j+C(\delta\gamma_j)^2]$$

$$A=\left(\frac{\partial^2\Psi}{\partial\Omega_i^{(2)}}\right)_{(2.12)}=A_1'-\mu_+=A_1'-\frac{A_3'}{\delta^2}+A_3'\frac{\omega_+}{\Omega\delta(1-\varepsilon)}=A_1'\left[\frac{\delta^2-1}{2\delta^2}+\frac{\delta^2+1}{2\delta^3(1-\varepsilon)}\frac{\omega_+}{\Omega}\right]$$

$$B=\left(\frac{\partial^2\Psi}{\partial\Omega_i\partial\gamma_i}\right)_{(2.12)}=-A_1'K_+=-A_1'\frac{\omega_+}{1-\varepsilon}=-A_3'\frac{2\omega_+}{(\delta^2+1)(1-\varepsilon)}$$

$$C=\left(\frac{\partial^2\Psi}{\partial\gamma_i^{(2)}}\right)_{(2.12)}=-(\nu_++K_+^2A_{*1})=mgc+A_3'\frac{\Omega\omega_+}{\delta(1-\varepsilon)}+A_{*3}\frac{\omega_+^2}{1-\varepsilon}-A_{*1}\frac{\omega_+^2}{(1-\varepsilon)^2}$$

$$\left(K_\pm=\frac{k\mp A_3'\Omega/\delta}{A_{*3}(\pm1-\varepsilon)^2}=\frac{\omega_\pm}{\pm1-\varepsilon}\right)$$

Hence, the conditions for solution (2.13) to be stable in the case when $\gamma_3=+1$ have the form $A>0, AC-B^2>0$, which is equivalent to the inequalities

$$\frac{\delta^2-1}{\delta^2+1}+\frac{\delta\omega_+}{\Omega(1-\varepsilon)}>0 \tag{2.15}$$

$$\left[\frac{\delta^2-1}{\delta^2+1}+\frac{\delta\omega_+}{\Omega(1-\varepsilon)}\right]\left[mgc+(A_{*3}(1-\varepsilon)-A_{*1})\frac{\omega_+^2}{(1-\varepsilon)^2}\right]+$$
$$+A_3'\frac{\delta^2-1}{\delta^2+1}\frac{\Omega\omega_+}{\delta(1-\varepsilon)}\left[1+\frac{\delta(\delta^2-1)}{\delta^2+1}+\frac{\omega_+}{\Omega(1-\varepsilon)}\right]>0 \tag{2.16}$$

The conditions for solution (2.13) to be stable in the case when $\gamma_3=-1$ have a form similar to conditions (2.15) and (2.16) with ω_+, ε, c, A_{*3} replaced by ω_-, $-\varepsilon$, $-c$, $-A_{*3}$ in them.

It follows from these conditions that rotation of the fluid which coincides in direction with the rotation of the rigid body ($\omega_\pm/\Omega>0$) is a stabilizing influence. If the envelope and the fluid rotate in different directions ($\omega_\pm/\Omega<0$), the region of stability is narrower than in the problem of the motion of a body without a fluid.

In particular, in the case of a spherical cavity ($\delta=1$) condition (2.15) and the analogous condition to it for $\gamma_3=-1$ denote that the fluid and the body rotate in the same direction, while condition (2.16) and the analogous condition for $\gamma_3=-1$ agree with the conditions of stability of the vertical rotations of the top corresponding to the case when there is no fluid [2].

3. THE CASE OF A WEIGHTLESS ENVELOPE

We will consider the case when the mass of the envelope can be neglected ($m_b=0, A_1=A_3=0$); in this case $A_{*3}=0$ and the generalized effective potential (2.11) has a singularity when $\gamma_1=\gamma_2=0$. Hence, the steady motions of the top with a weightless envelope will be investigated by direct analysis of the critical points of function (2.1) at fixed levels of the first integrals (2.2)–(2.4).

We will represent the non-increasing function H and the generalized Jellet integral J in the form ($m_l=m$)

$$H=(ma_3^2/5)H_0,\quad J=(ma_3^2/5)J_0$$

where

$$H_0=\frac{5}{2a_3^2}(v_1^2+v_2^2+v_3^2)+\frac{1}{2}\left[\frac{(\delta^2-1)^2}{\delta^2+1}(\omega_1^2+\omega_2^2)+\right. \tag{3.1}$$

$$+4\frac{\delta^2}{\delta^2+1}(\Omega_1^2+\Omega_2^2)+2\delta^2\Omega_3\Bigg]-G\gamma_3\leq h_0,\quad G=5\frac{gc}{a_3^2}$$

$$J_0=\frac{(\delta^2-1)^2}{\delta^2+1}(\omega_1\gamma_1+\omega_2\gamma_2)+4\frac{\delta^2}{\delta^2+1}(\Omega_1\gamma_1+\Omega_2\gamma_2)+2\delta^2\Omega_3\gamma_3=k_0 \tag{3.2}$$

To find the critical points of function (3.1) with conditions (3.2), (2.3) and (2.4) we will introduce the function

$$V=H_0-\lambda(J_0-k_0)-\frac{1}{2}\mu(W-\Omega^2)-\frac{1}{2}\nu(\Gamma-1)$$

where λ, μ, ν are Lagrange multipliers, and we will write the conditions for it to be stationary with respect to the variables $\mathbf{v}$, $\boldsymbol{\omega}$, $\boldsymbol{\Omega}$, $\boldsymbol{\gamma}$, λ, μ and ν. As a result we obtain

$$\begin{aligned}
&\frac{\partial V}{\partial v_i}=\frac{5}{a_3^2}v_i=0,\quad i=1,2,3\\
&\frac{\partial V}{\partial \omega_j}=\frac{(\delta^2-1)^2}{\delta^2+1}(\omega_j-\lambda\gamma_j),\quad \frac{\partial V}{\partial \Omega_j}=4\frac{\delta^2}{\delta^2+1}(\Omega_j-\lambda\gamma_j)-\mu\Omega_j=0\\
&\frac{\partial V}{\partial \gamma_j}=-\lambda\frac{(\delta^2-1)^2}{\delta^2+1}\omega_j+4\lambda\frac{\delta^2}{\delta^2+1}\Omega_j-\nu\gamma_j=0;\quad j=1,2\\
&\frac{\partial V}{\partial \Omega_3}=\delta^2[(2-\mu)\Omega_3-2\lambda\gamma_3]=0\\
&\frac{\partial V}{\partial \gamma_3}=-G-2\lambda\delta^2\Omega_3-\nu\gamma_3=0\\
&\frac{\partial V}{\partial \lambda}=-(J_0-k_0)=0,\quad \frac{\partial V}{\partial \mu}=-(W-\Omega^2)=0,\quad \frac{\partial V}{\partial \nu}=-(\Gamma-1)=0
\end{aligned} \tag{3.3}$$

Obviously system (3.3) allows of the solutions

$$\begin{gathered}
v_1=v_2=v_3=\omega_1=\omega_2=\Omega_1=\Omega_2=\gamma_1=\gamma_2=0,\quad \gamma_3=\pm1,\quad \Omega_3=\Omega/\delta\\
(\lambda=\mp p\Omega\ (p\in\mathbf{R}),\quad \mu=2(1-p\delta),\quad \nu=\mp G-2p\delta\Omega^2)
\end{gathered} \tag{3.4}$$

similar to solutions (2.13) with corresponding upper and lower signs.

To investigate the stability of the steady motions (3.4) we will calculate the second variation of the function V on the linear manifold $\delta J_0=\delta W=\delta\Gamma=0$, which for these solutions has the form (2.14), retaining for the variations of the variables $v_i (i=1,2,3)$, ω_j, Ω_j, γ_j $(j=1,2)$ their previous notation. As a result we obtain

$$\begin{aligned}
\delta^2V=&\frac{5}{2a_3^2}(v_1^2+v_2^2+v_3^2)+\frac{1}{2}\sum_{j=1}^{2}\left[\frac{(\delta^2-1)^2}{\delta^2+1}\omega_j^2+2\left(p\delta+\frac{\delta^2-1}{\delta^2+1}\right)\Omega_j^2+\right.\\
&\left.+(\pm G+2p\delta\Omega^2)\gamma_j^2-2p\Omega\frac{(\delta^2-1)^2}{\delta^2+1}\omega_j\gamma_j-8\frac{\delta^2}{\delta^2+1}p\Omega\Omega_j\gamma_j\right]
\end{aligned} \tag{3.5}$$

(the plus sign in front G is taken for solution (3.4) when $\gamma_3=+1$, and the minus sign is taken for solution (3.4) when $\gamma_3=-1$).

Quadratic form (3.5) is positive definite when

$$\frac{\delta^2-1}{\delta^2+1}+p\delta>0 \tag{3.6}$$

$$\Omega^2\frac{\delta^2-1}{\delta^2+1}f(p)\pm\left(\frac{\delta^2-1}{\delta^2+1}+p\delta\right)G>0 \tag{3.7}$$

$$f(p)=-p^3\delta(\delta^2-1)+p^2(\delta^2-1)+2p\delta$$

Taking into account the arbitrariness of the parameter p, we conclude that the steady motions (3.4) are stable if $p \in \mathbb{R}$, for which inequality (3.6) and the corresponding inequality (3.7) with the upper or lower signs are simultaneously satisfied. We will analyse inequality (3.6) and inequality (3.7) with the upper sign. If $\delta > 1$, these inequalities are satisfied when $p = 0$. If $\delta < 1$, inequality (3.6) is satisfied when

$$p>p^0>0 \quad \left(p^0=\frac{1-\delta^2}{\delta(1+\delta^2)}\right)$$

For these values of p the coefficients of Ω^2 on the left-hand side of inequality (3.7) with the upper sign are negative, and this inequality is only satisfied when $\Omega^2 < \Omega_p^2$, where

$$\Omega_p^2=G\frac{F(p)}{p^0},\quad F(p)=\frac{p-p^0}{f(p)}\quad (p>p^0)$$

Hence, when $\delta \in (0, 1)$ inequality (3.6) and inequality (3.7) with the upper sign are satisfied simultaneously when $p = p_0 > p^0$, if

$$\Omega^2<\Omega_0^2=GF(p_0)/p^0 \tag{3.8}$$

where p_0 is the maximum point of the function $F(p)$ on the ray $(p^0, +\infty)$.

We can similarly analyse inequality (3.6) and inequality (3.7) with the lower sign. When $\delta > 1$ these inequalities are satisfied simultaneously when

$$p=p_1\in(p',0),\quad \text{if}\quad \Omega^2>\Omega_1^2=GF(p_1)/p^0 \tag{3.9}$$

where p' is the smaller root of the equation $f(p) = 0$ and p_1 is the minimum point of the function $F(p)$ in the interval $(p', 0)$. When $\delta \in (0, 1)$ these inequalities are not satisfied simultaneously for any $p \in \mathbb{R}$.

Hence, the stability of the rotations of a top with a liquid filling around a vertical axis of symmetry on a horizontal plane with friction depends very much (in the case of weightless envelope) both on the arrangement of the cavity and on its shape. If the centre of the cavity is below the centre of the spherical envelope, the rotation of the top (solution (3.4) for the case $\gamma_3 = +1$) is always stable if the cavity is an oblate spheroid ($\delta > 1$), and stable for a small vorticity of the fluid (see (3.8)), if the cavity is a prolate spheroid ($0 < \delta < 1$). If the centre of the cavity is below the centre of the spherical envelope, rotation of the top (solution (3.4) for the case $\gamma_3 = -1$) is always unstable for $0 < \delta < 1$ and for $\delta > 1$ is stable for large fluid vorticity (see (3.9)).

In particular, if the centre of the cavity coincides with the centre of the weightless spherical envelope (in this case $G = 0$), uniform rotation of the top around a vertical axis of symmetry on a horizontal plane with friction is always stable (unstable) if the cavity is oblate, $\delta > 1$ (prolate, $\delta < 1$) along the axis of symmetry. Hence, the presence of sliding friction disturbs the stability of the top rotations on an absolutely smooth plane [3] in the case of a strongly prolate cavity ($\delta < 1/3$).

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REFERENCES

1. MOISEYEV, N. N. and RUMYANTSEV, V. V., Dynamics Stability of Bodies Containing Fluid. Springer, Berlin, 1968.
2. KARAPETYAN, A. V., The Stability of Steady Motions. Editorial URSS, Moscow, 1998.
3. MARKEYEV, A. P., The stability of the rotation of a top filled with fluid. Izv. Akad Nauk SSSR, MTT, 1985, 3, 19–26.

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